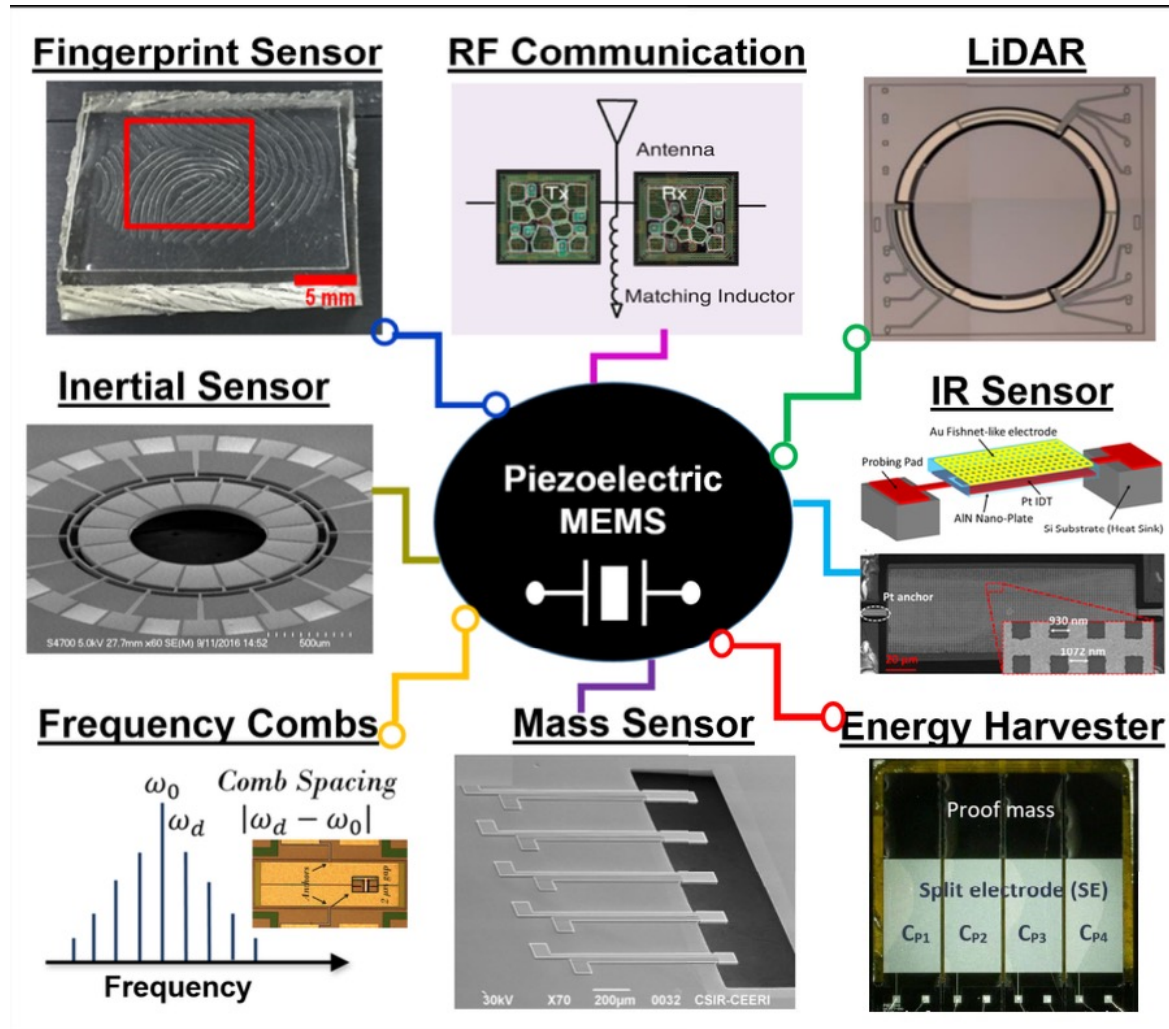
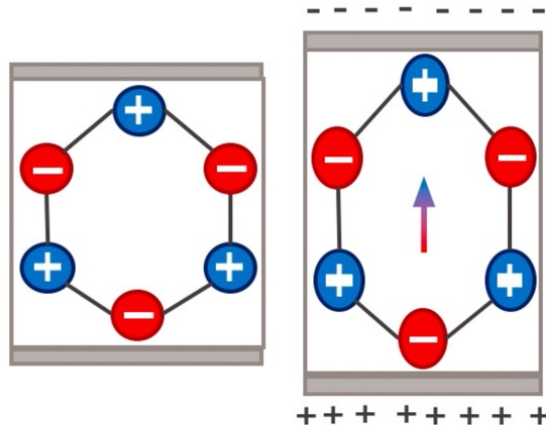




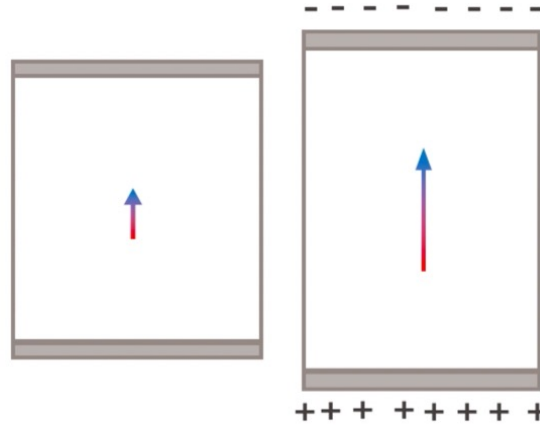
G. Pillai and S.-S. Li,
 "Piezoelectric MEMS Resonators: A Review,"
IEEE Sensors Journal, vol. 21, no. 11, pp. 12589–12605, Jun. 2021
 doi: [10.1109/JSEN.2020.3039052](https://doi.org/10.1109/JSEN.2020.3039052)

Piezoelectric Effect

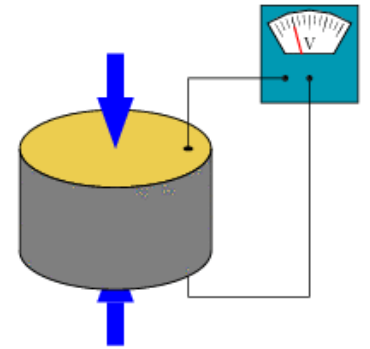
- Piezoelectricity
 - Material property through which:
 - An applied electric field leads to mechanical strain
 - Mechanical stress generates charges
 - It occurs in materials without inversion symmetry
 - Either the Unit lattice is not symmetric, or ferroelectric material



cristaux
non-centrosymétriques



matériaux
ferroélectriques



<https://en.wikipedia.org/wiki/Piezoelectricity>

Both ways! Direct and reverse

- Motion when apply a voltage
- Generate voltage/current when impose motion

Piezoelectric Actuation

- Examples of piezo materials:

- PZT: $\text{Pb}(\text{Zr,Ti})\text{O}_3$
- Quartz
- AlN
- AlScN AlScN (depends on Sc content)
- ZnO
- LiNbO_3
- PMN-PT: $0.7 \text{ Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3 - 0.3 \text{ PbTiO}_3$
- PVDF $\text{Poly}(1,1\text{-difluoroethylene})$

Black: crystal non-centrosymmetric
Red: ferroelectric

- Fast (up to GHz)
- Linear in strain-voltage and stress-voltage
- High efficiency
- High Young's modulus (of order 60 GPa except for polymers)
- Small displacements (strain typically 0.1%)
- Highest actuator performance materials (PZT) contain Pb (Lead). Challenging to find good Pb-free options for actuation.

Piezoelectric effect: links deformation (i.e. strain, hence stress) and electric polarization

- An applied Electric field leads to mechanical strain
- Mechanical stress generates charges

$$\varepsilon_{ij} = S_{ijkl} \sigma_{kl} + d_{ijk} E_k$$

$$D_i = \epsilon_{ij} E_j + d_{ijk} \sigma_{jk}$$

Purple: all materials

Green: only piezo

S_{ijkl} Compliance tensor (inverse of Elasticity tensor)

ε_{ij} Strain (if single index: 1,2,3: axial, 4,5,6: shear)

σ_{kl} Stress (if single index: 1,2,3: axial, 4,5,6: shear)

ϵ_{ij} Permittivity matrix

E_k Electric field components (not Young's modulus!)

D_i Displacement field (electric)

d_{ijk} Piezo tensor

$$\vec{D} = \epsilon \vec{E} + \vec{P}$$

$\vec{P}$ Polarization: density of electric dipoles.
units: C/m²

Piezo matrix links charge and stress (or voltage and strain)

The displacement field D is equivalent to a surface charge, C/m^2

The surface charge density is equal to the remanent internal polarization

$$D_i = d_{ijk} \sigma_{kl}$$

The units of the piezoelectric coefficients d_{ij} is C/N (i.e. Coulomb per Newton), or m/V (i.e. Meter per Volt)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & \dots & \dots & \dots & d_{16} \\ d_{12} & \dots & \dots & \dots & \dots & \dots \\ \underbrace{d_{31} \quad d_{32} \quad \dots}_{axial} & \underbrace{\dots \quad \dots \quad d_{36}}_{shear} \end{pmatrix} \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{pmatrix}$$

Charge per unit area stress

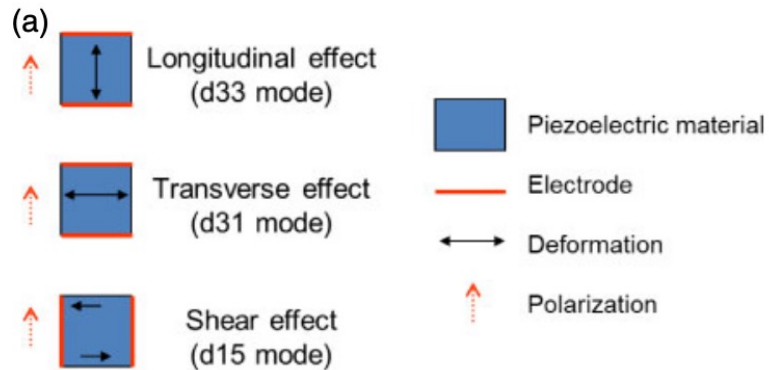
$$\begin{pmatrix} \varepsilon_x \end{pmatrix} = \begin{pmatrix} d_{11} \\ d_{31} \end{pmatrix}^t \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

The piezo matrix has 18 components (6x3)

- The left side of the matrix represents piezoelectric coefficients linked to **axial** deformations
- The right side of the matrix represents piezoelectric coefficients linked to **shear** deformations

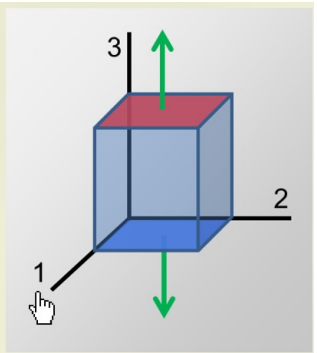
$$d_{ik}$$

k = direction of mechanical action
 i = direction of the electric field
 = direction perpendicular to the electrodes



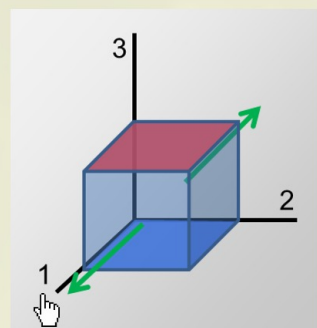
Compression Mode

d_{33} would indicate the polarization generated in the 3-direction per unit of mechanical compression stress (T) applied in the 3-direction to the piezoelectric body.



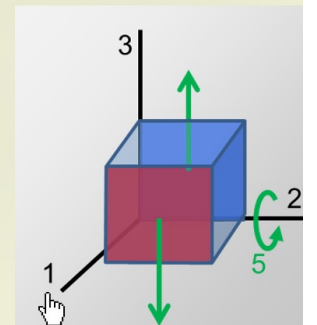
Transverse Mode

d_{31} is the polarization developed in the 3-direction per unit stress applied in the 1-direction (all other external stresses = 0).



Shear Mode

d_{15} means the polarization developed in the 1-direction per unit shear stress 5 applied (= shear around the 2-direction) when there are no other external stresses.



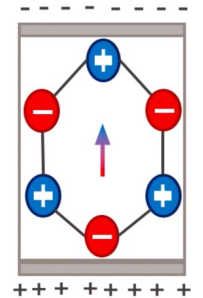
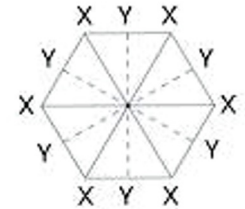
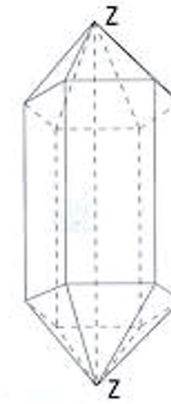
Piezo matrix for quartz

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} d_{11} & -d_{11} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} + \begin{pmatrix} d_{14} & 0 & 0 \\ 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}$$

Five non-zero coefficients and only 2 values (due to symmetry)

$$d_{11} = 2.3 \cdot 10^{-12} \text{ C/N}$$

$$d_{14} = -0.7 \cdot 10^{-12} \text{ C/N}$$



- When using a piezo element as a sensor, C/N (charge per applied force) is more convenient (usually pC/N)
- When using a piezo as an actuator, m/V is more intuitive (usually pm/V)
- C/N = m/V

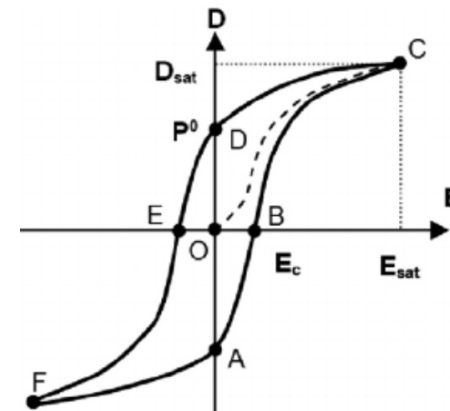
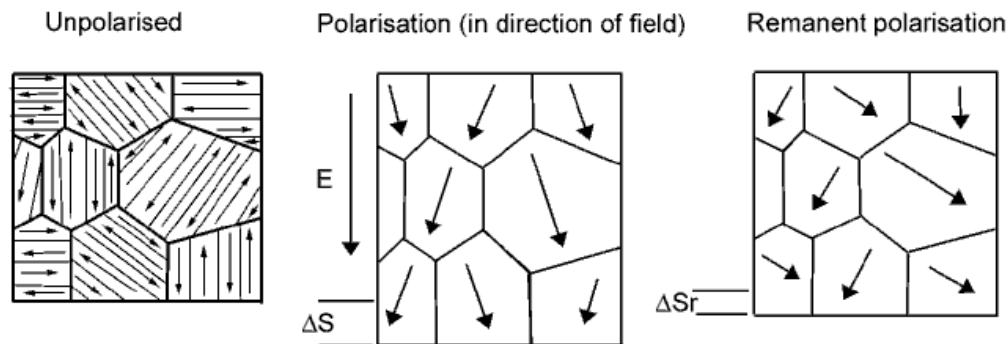
But Quartz is not easily MEMS compatible...

Ferroelectric materials

In a ferroelectric material, the dipoles are coupled

The coupling of the dipoles makes them line up with each other, creating ferroelectric domains. One can impose polarization by applying an external field and heating.

Curie Temperature T_C : above which the spontaneous polarization disappears. Need to stay below T_C unless wish to remove the polarization



Piezo Matrix for piezo-ceramics

- Ferroelectric ceramics must be polarized (poled) to become piezoelectric.
- Polarization is obtained by applying a high electric field (typically just below breakdown) at high temperature, typically the final fabrication step.
- The Polarization direction is termed the z axis (by convention)

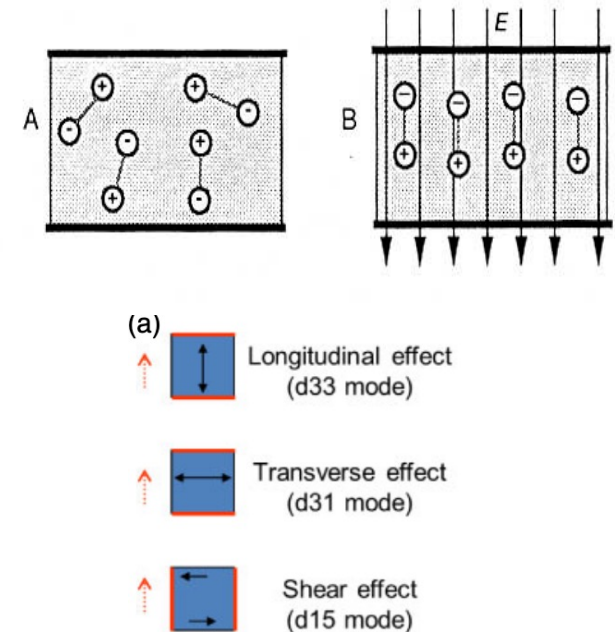
$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \cdot (\sigma_{1...6})$$

Five non-zero coefficients and only 3 different values

For PZT:

$$d_{33} = 375 \cdot 10^{-12} \text{ C/N} \quad d_{31} = -190 \cdot 10^{-12} \text{ C/N} \quad d_{15} = 515 \cdot 10^{-12} \text{ C/N}$$

The relative permittivity of ceramic piezoelectric materials is very high: $\epsilon_r = 1200$



Electromechanical Coupling coefficient k

- k_{ij} denotes the coupling coefficient between the mechanical energy input in the i -direction and the energy converted in the j -direction, or vice-versa.
- k is an indicator of the effectiveness with which a piezoelectric material converts electrical energy into mechanical energy, or converts mechanical energy into electrical energy.

$$k^2 = \frac{\text{converted energy}}{\text{input energy}}$$

- $k=0$ No coupling
- $k=1$ Perfect coupling
- k is less than one because of losses (dielectric, mechanical, acoustic), boundary clamping, imperfect materials, non-uniform fields

Comparing Piezo materials

1. Non-ferroelectric materials: Quartz (T.=570 °C)

Advantages

- Mechanical robustness
- Linearity, no hysteresis
- Can be used up to 400° C (transition temperature 573° C)
- No pyroelectric effect (no T dep)
- No aging

Drawbacks

- Price (~10'000 x more expensive than PZT)
- Requires a charge amp (weak couplings)
- Difficult to assemble

- good for precision devices like oscillators.
- But can't integrate Quartz in MEMS flow (AlN however OK)

2. Ferroelectric materials: PZT ceramics (Lead - Zirconium - Titanium, + oxides), BaTiO, Single crystals: LiNbO, (T=1210 °C),...

Advantages

- low cost
- High piezoelectric modules
- High electrical permittivity (1200) → High capacitance
- Machining/casting possible
- Adjustable Curie temperature (depending on composition)

Drawbacks

- Thermal dependence of coefficients
- Thermal hysteresis

- good for applications requiring high energy density.
- Can bond thin films to wafer

3. Polymer piezo materials

Polyvinylidene Fluoride (PVDF) is the most widely used piezoelectric polymer. The piezoelectric effect in PVDF arises from its molecular structure. $d_{33} = -33$ pC/N.

- Lightweight, flexible
- Can be cast as thin film

Advantages:

- **Flexibility and Processability:** can be manufactured into large areas, thin sheets and various shapes, offering design flexibility.
- **High Piezoelectric Stress Constants:** PVDF has higher piezoelectric stress constants (g_{33} , units of V. m /N) than ceramics like PZT, ie better sensitivity in sensor applications.

$$g_{33} = \frac{d_{33}}{\epsilon_0 \epsilon_r}$$

Drawbacks:

- Pyroelectric effect (leads to thermal drift)
- Low Curie temperature (120 °C)
- Mechanical damping
- Lower Electro-Mechanical Coupling Factor

Property	PVDF (Polyvinylidene Fluoride)	PZT (Lead Zirconate Titanate)	Quartz	BaTiO ₃ (Barium Titanate)
Density (g/cm ³)	1.78-1.80	7.5-8.0	2.65	~6.0
Dielectric Constant	10-12	300-1200	~4.5	1200-1700
d_{33} (pC/N)	-20 to -33	200-600	~2.3	~190-350
Pyroelectric Constant (μC/m ² K)	~30	~300	~0.1	~100
Elastic Modulus (GPa)	2-4	50-70	~80	~30-35
Electromechanical Coupling Constant (k ²)	~0.12	0.35-0.7	~0.1	~0.45-0.5

Source: ChatGPT, 12-2024

EPFL Piezoelectric Actuation (linear)

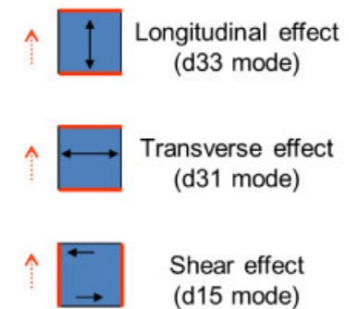
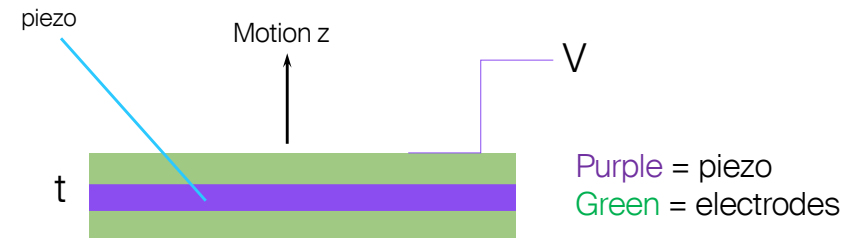
- An electric field leads to mechanical strain, and to motion of amplitude z
- Direct expansion $\rightarrow z = d_{33}V$

Derivation from strain

$$\epsilon_z = d_{33}E_z = \frac{d_{33}V}{t}$$
$$z = \epsilon_z t = d_{33}V$$

- d_{33} in $\left[\frac{pm}{V}\right]$, $AlN \rightarrow \frac{6pm}{V}$, $PZT \rightarrow 100 \text{ to } 200 \frac{pm}{V}$
- z-motion has No Dependence on film Thickness !
 - actuation displacement and force scale with V , not V/t
 - different from capacitive actuation (scales as E^2)

- z scales as L^0 (if assume V scale invariant).
- z scales as L^1 if assume V limited by breakdown

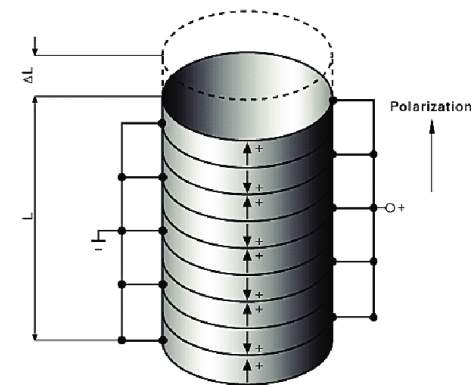
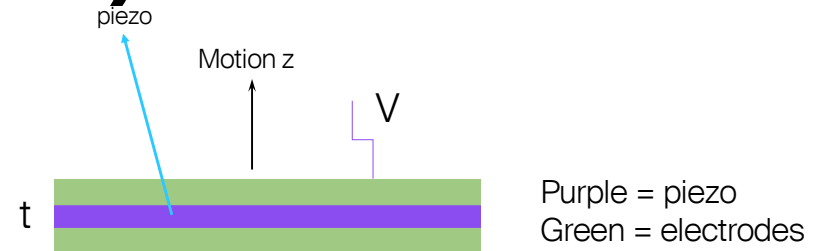


Piezoelectric Actuation (linear)

- Voltage is limited by breakdown voltage (or by power supply). Often up to 200V
- Output force links stress and strain via Y (Young's modulus).
 - $F = P A = A Y . \epsilon_z = A Y \frac{d_{33} V}{t} = A Y d_{33} E$
- F scales as L^2
- To get higher displacement: use multilayers. This is very common in cm-scale devices

- Multilayer: $z = d_{33} V * N$

$$\text{Energy density} = \frac{1}{2} \epsilon^2 = \frac{1}{2} \left[\frac{d_{33} V}{t} \right]^2 \propto L^{-2} \text{ or } L^0$$

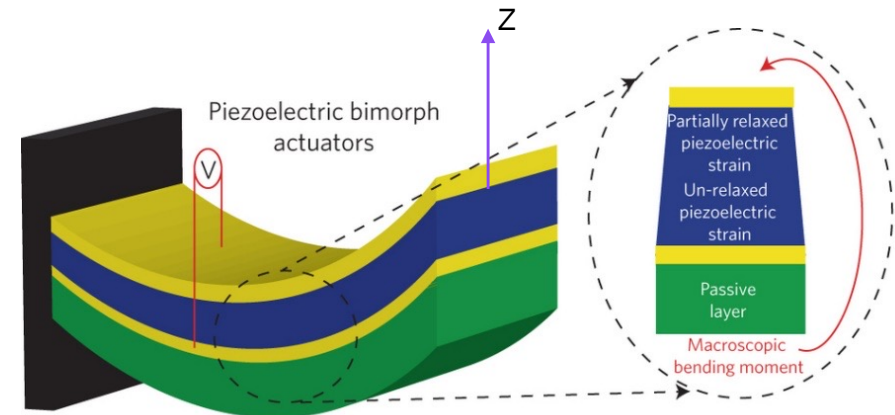
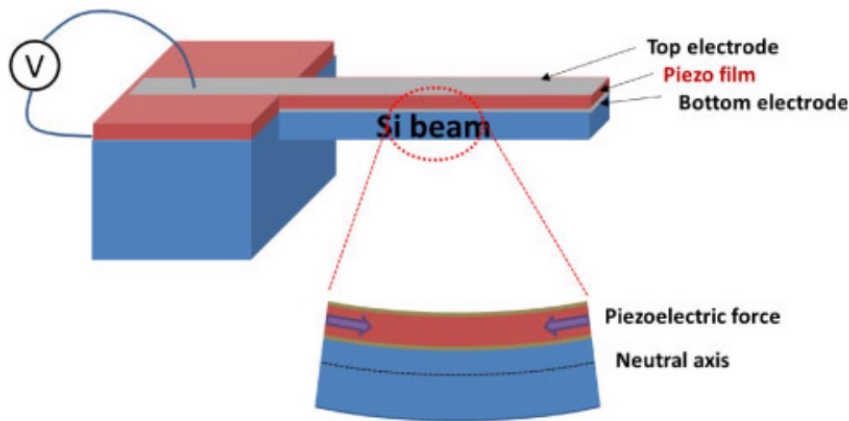
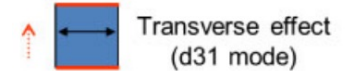


https://www.researchgate.net/figure/Structure-of-a-PZT-stack3_fig3_271882331



<https://piezodirect.com/multilayer-piezo-actuators/>

Piezoelectric Actuation (bending)



Kanno, I. Piezoelectric MEMS: Ferroelectric thin films for MEMS applications. *Jpn. J. Appl. Phys.* **57**, 040101 (2018).
<https://iopscience.iop.org/article/10.7567/JJAP.57.040101>

- Make bending bimorph: piezo layer expands, the beam bends
- Coupling through bending moment $\rightarrow z = \chi_A \frac{z_{offset}}{t} \left(\frac{L}{t}\right)^2 d_{31} V = \chi_A z_{offset} \frac{L^2}{t^3} d_{31} V$

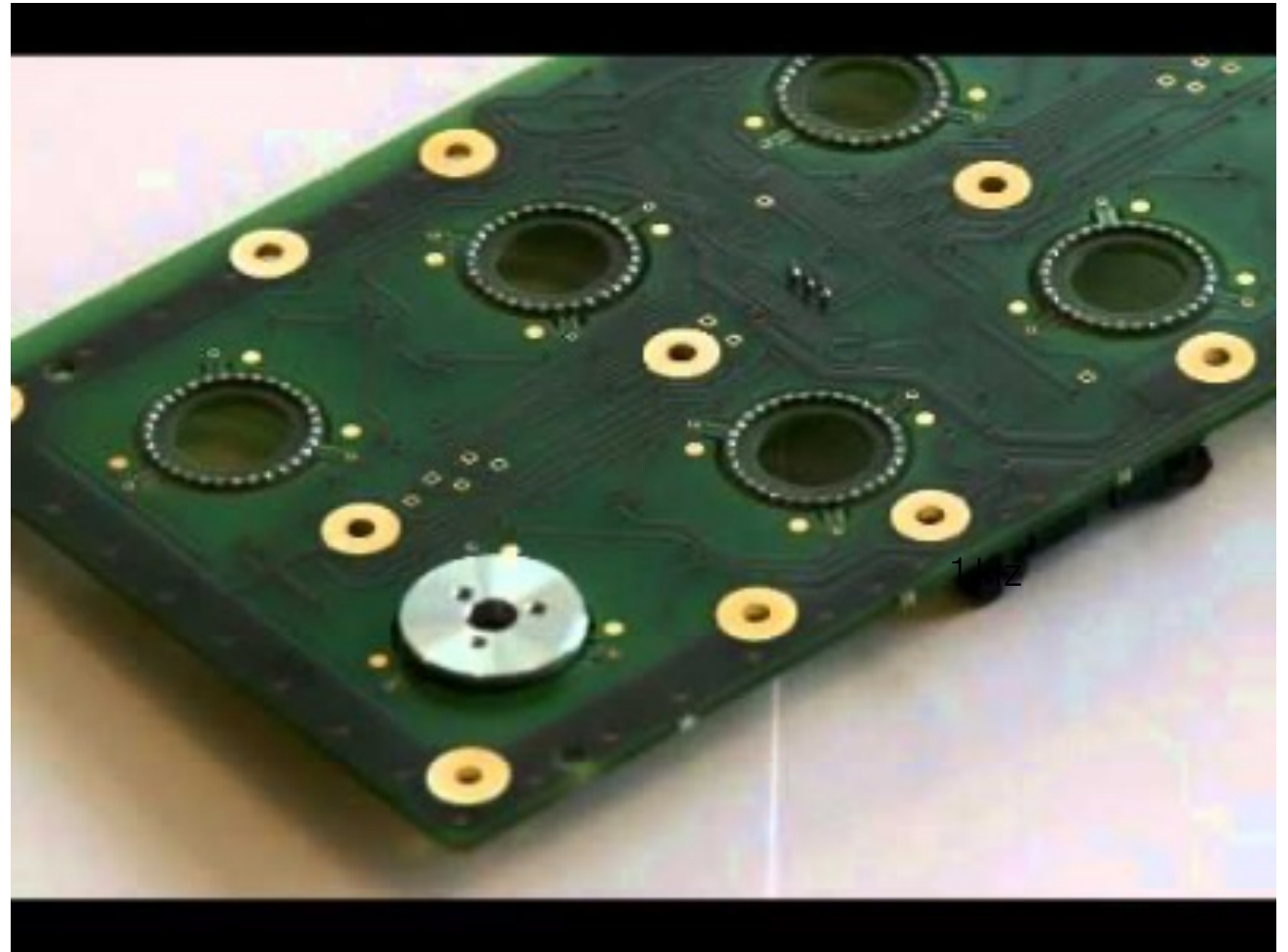
Assumes: all layers have the same Young's modulus (so E disappears from equations)

z_{offset} : distance from beam neutral axis to center of piezo films

Z Scales as L^0 or
as L^1 (if V scales as L)

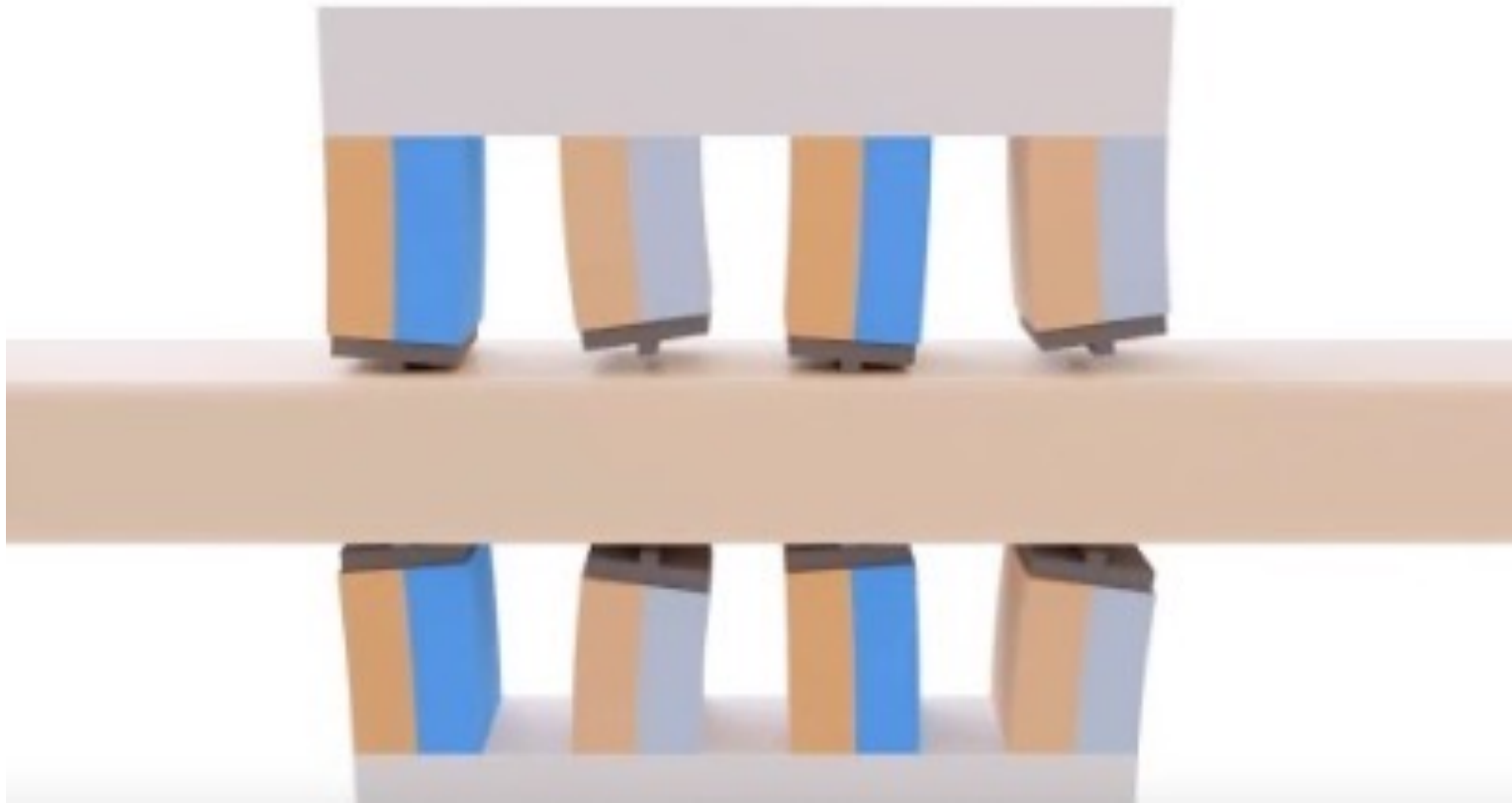
cm-scale motion, from μm scale piezo motion

ultrasonic piezoelectric
motors



<https://www.youtube.com/watch?v=VbTUsluY2xU>

Piezomotor, mm-scale (or less)

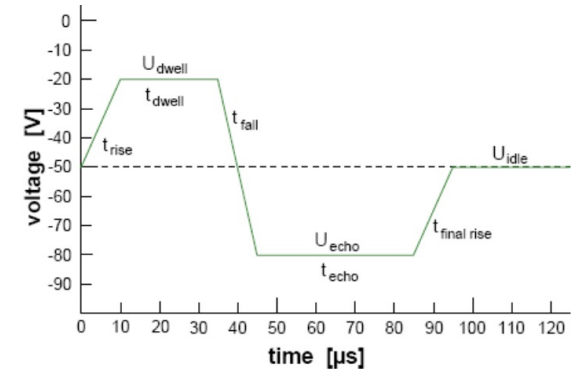
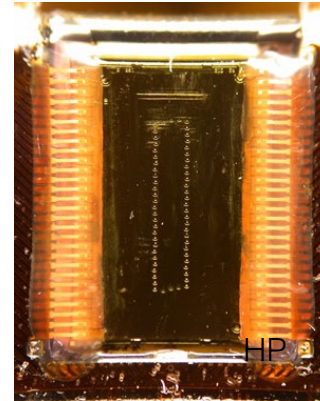
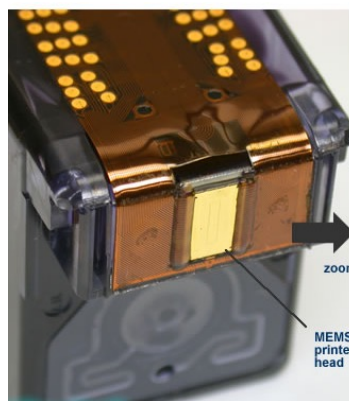


<https://www.youtube.com/watch?v=7iHL4ZCkCKc>

nm steps at many kHz

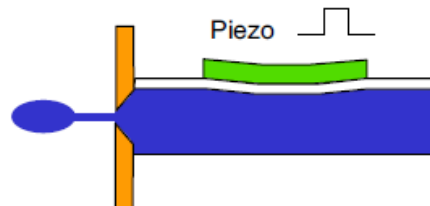
<https://www.piezomotor.com/>

Piezoelectric Actuation at mm scale, μm motion: inkjet printing



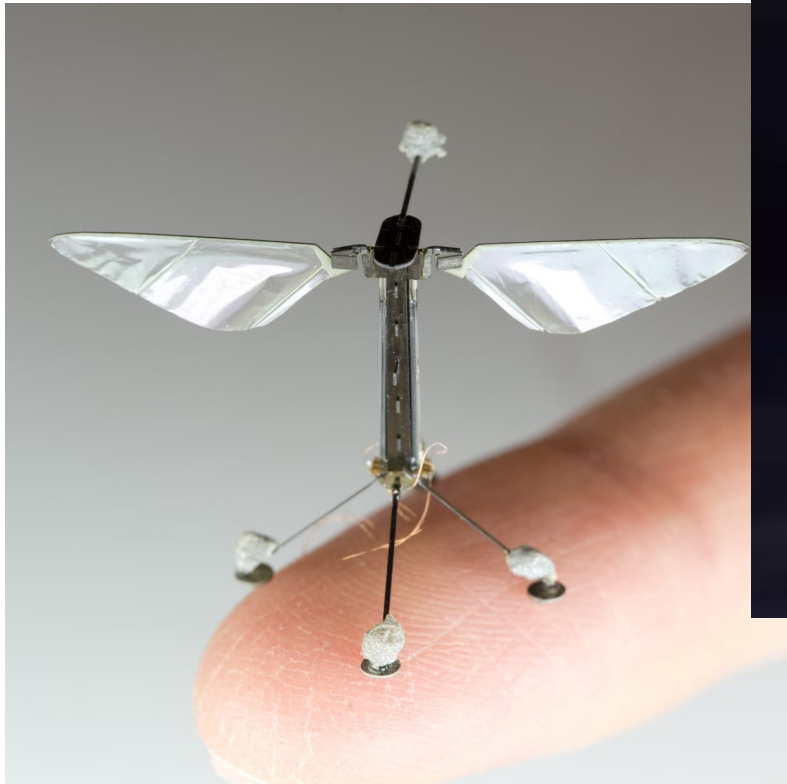
Inkjet printheads – Drop on Demand (DoD)
http://microfab.com/index.php?option=com_content&view=article&id=93&Itemid=9

Piezo



■ Piezo crystal deforms when electrical pulse applied – many different architectures

Operation at >10 kHz



<https://vimeo.com/65313515>

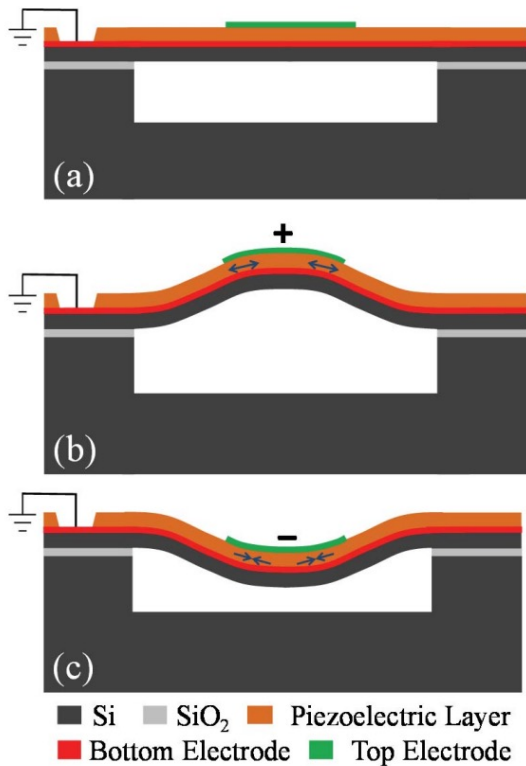
Rob Wood and co, Robobee, Harvard University

Piezo bimorph (2 antagonistic), 200 Hz

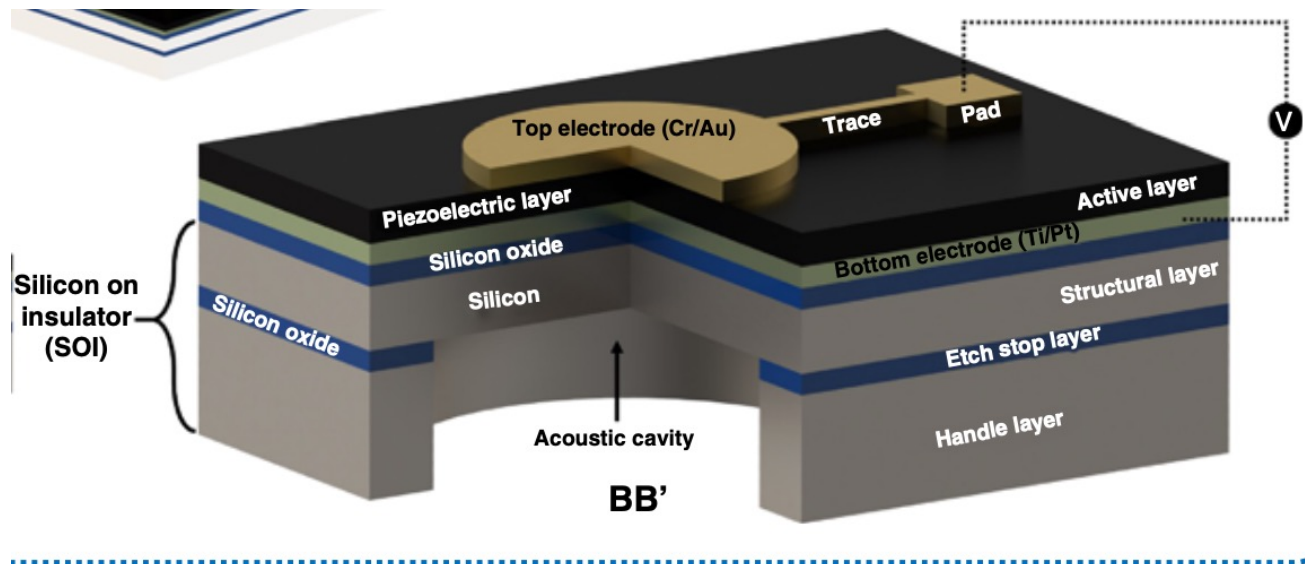
K. Y. Ma, P. Chirarattananon, S. B. Fuller, and R. J. Wood, "Controlled Flight of a Biologically Inspired, Insect-Scale Robot," *Science*, vol. 340, no. 6132, pp. 603–607, May 2013, doi: [10.1126/science.1231806](https://doi.org/10.1126/science.1231806).

EPFL Ultrasound sources (PMUT)

piezoelectric micromachined ultrasonic transducer. μm -scale



Y. Lu and D. A. Horsley, JMEMS 2015,
doi: [10.1109/JMEMS.2014.2387154](https://doi.org/10.1109/JMEMS.2014.2387154).



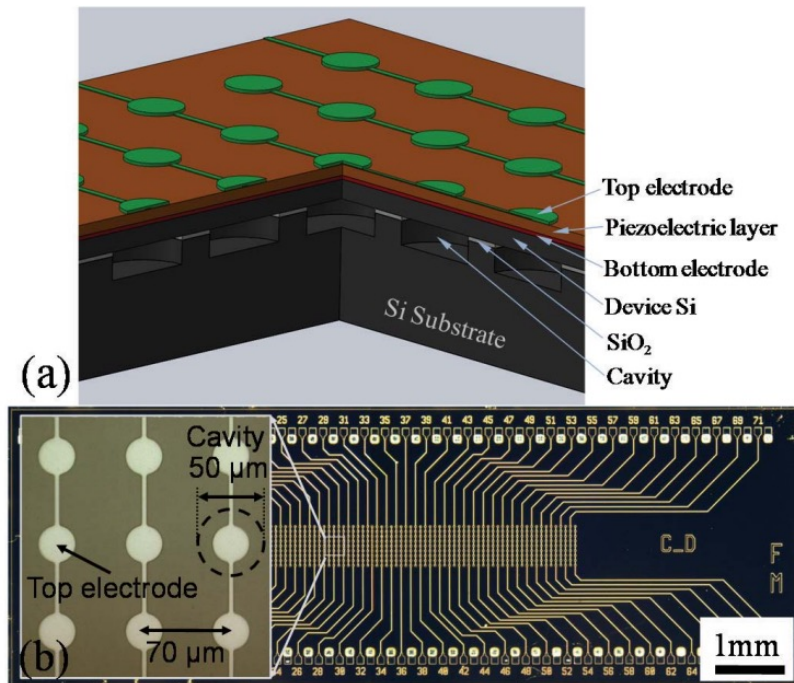
Roy et al, Microsystems and
Nanoengineering 2023
doi: [10.1038/s41378-023-00555-7](https://doi.org/10.1038/s41378-023-00555-7).

PMUT: Must both sense **and** actuate

Scales up to loudspeakers

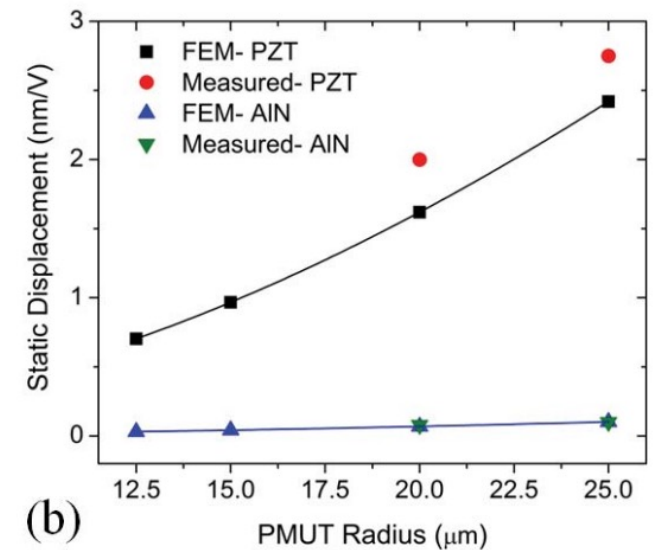
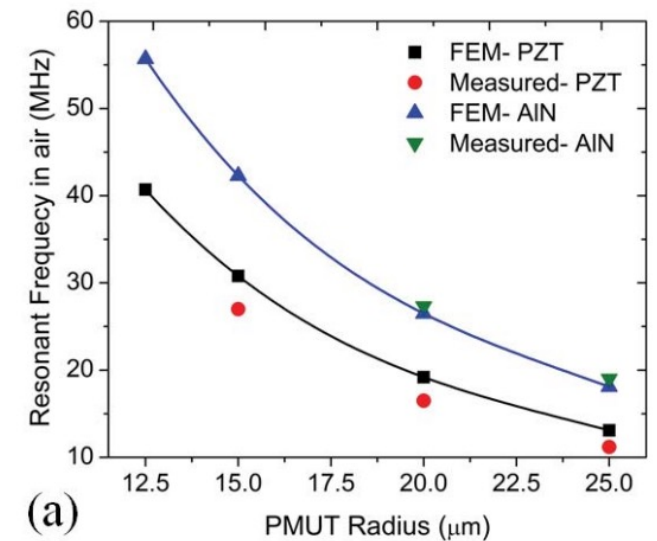
Ultrasound sources (PMUT)

piezoelectric micromachined ultrasonic transducer. μm -scale



PMUT: Must sense and actuate

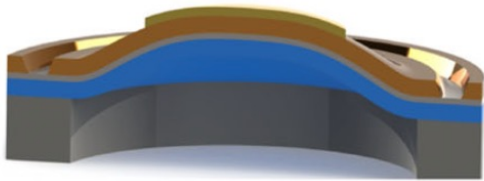
13-55 MHz



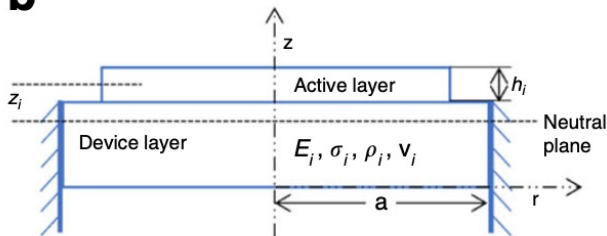
Scales up to loudspeakers

PMUT structural classification

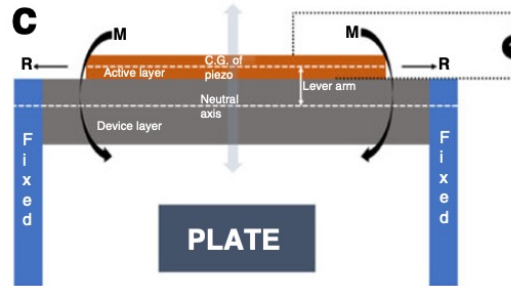
a



b



c

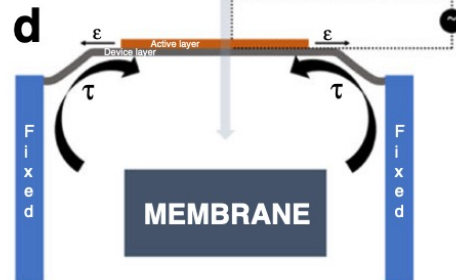


Flexural rigidity (D_e) dominated

$$D_e = \sum_i \left(\frac{E_i}{1 - \nu_i^2} \right) \left(\frac{h_i^3}{12} + z_i^2 h_i \right)$$

E_i – Young's Modulus of the i^{th} layer
 ν_i – Poisson's ratio of the i^{th} layer
 h_i – Thickness of i^{th} layer
 z_i – Distance of the i^{th} layer from the neutral axis

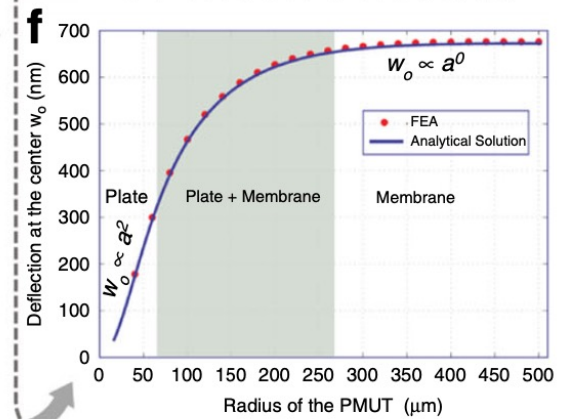
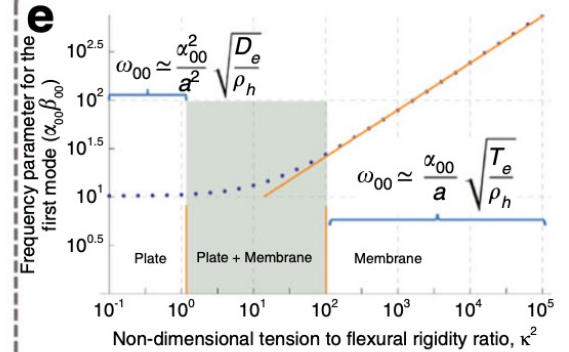
d



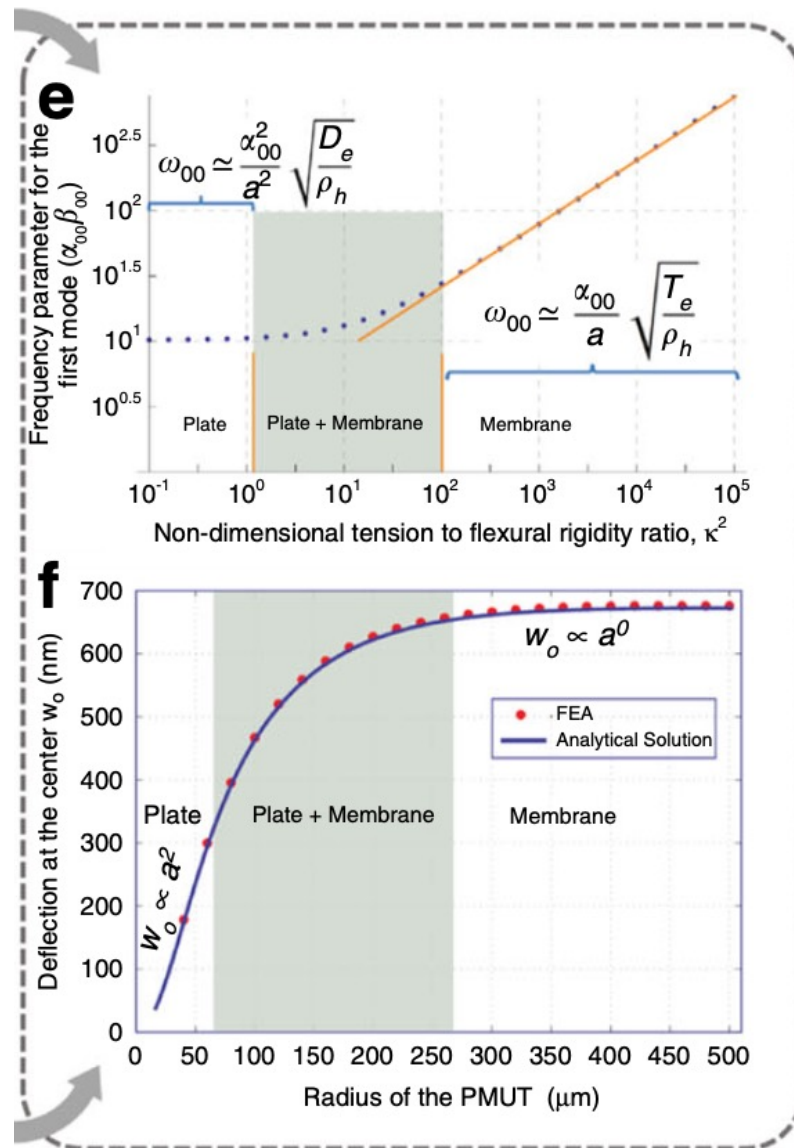
Pretension (T_e) dominated

$$T_e = \sum \sigma_i h_i$$

σ_i – Prestress of the i^{th} layer



Roy et al, Microsystems and Nanoengineering 2023
 doi: 10.1038/s41378-023-00555-7.



Roy et al, Microsystems and
Nanoengineering 2023
doi: 10.1038/s41378-023-00555-7.

RF FILTERS

- For acoustic resonators, the mechanical wavelength in materials (μm scale at GHz) is several orders of magnitude smaller than the electromagnetic wavelength (cm scale at GHz in air)
 - Good fit for MEMS device
- RF Filters using piezos:
 - SAW: surface acoustic wave (< 2 GHz)
 - BAW: bulk acoustic wave

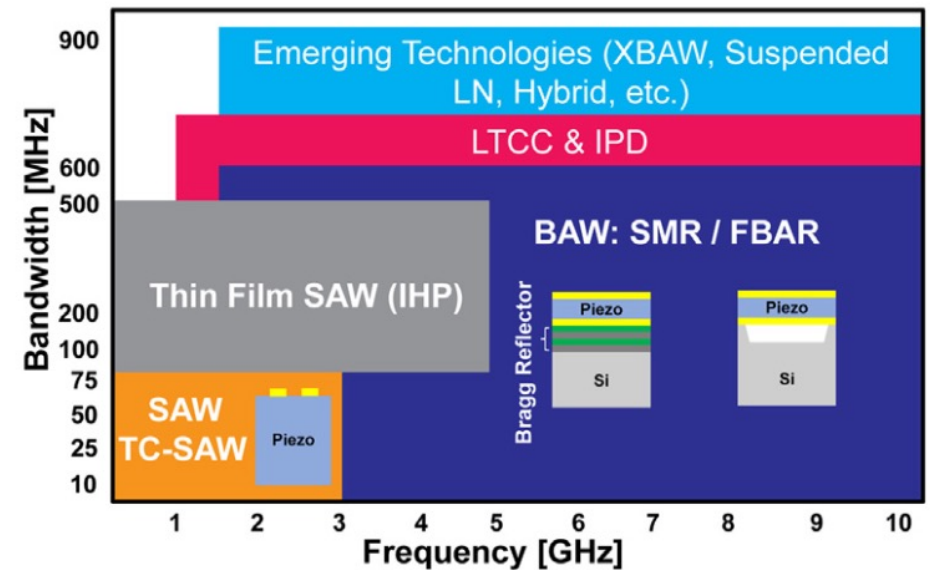


Fig. 2 | Advanced RF filters for wireless communications.

K. Yang, C. He, J. Fang, X. Cui, H. Sun, Y. Yang, C. Zuo, Advanced RF filters for wireless communications. *Chip* 2, 100058 (2023).

SAW: surface acoustic wave (< 2 GHz)

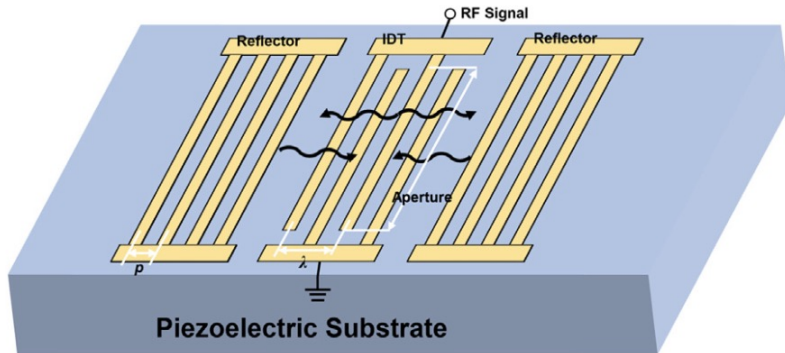
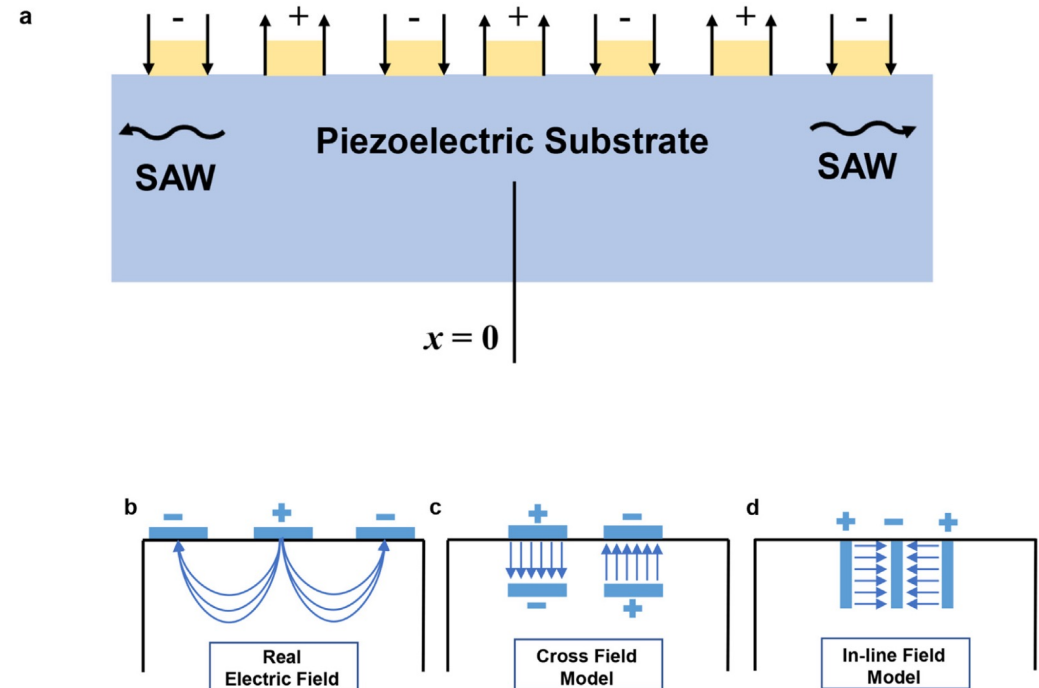


Fig. 3 | Typical structure of a SAW resonator.

$$f_0 = \frac{V_{SAW}}{\lambda}$$

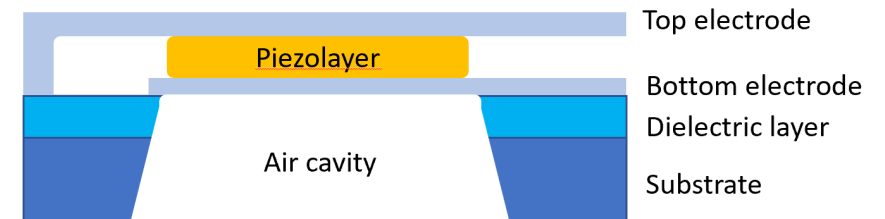
- λ is the wavelength, given by the periodicity of the IDT
- V_{SAW} : acoustic wave velocity.

$\lambda/4$ spacing between fingers defines frequency:
lithographically defined.



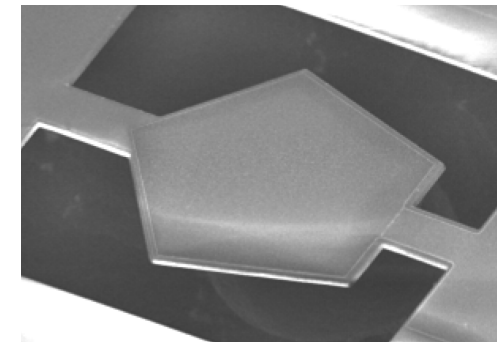
Piezoelectric resonator

- Use Piezoelectric effect to deform resonator... and read also out the motion
- Bulk Acoustic Resonator (BAR)
 - Acoustic waves constructive interaction
 - *Typical frequencies:* 100 MHz – 10 GHz
 - *Applications:* Filters, Oscillators
- *Advantages:*
 - High frequency, high Q
 - Easy electromechanical modelling
- *Disadvantages:*
 - Complicated design (to trim properties)
 - One frequency per wafer
- Piezo scales well down to small sizes: very effective to drive oscillators and filters
- Much lower voltage than capacitive MEMS. Direct drive with CMOS levels



https://en.wikipedia.org/wiki/Thin-film_bulk_acoustic_resonator

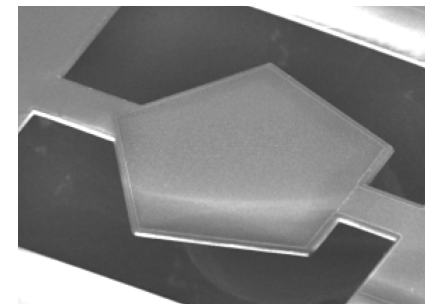
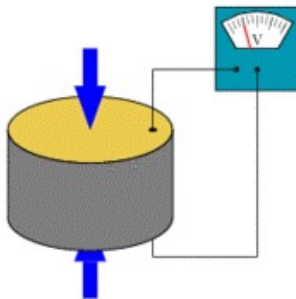
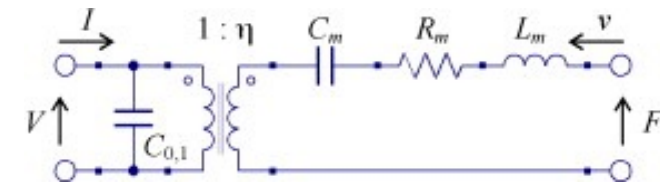
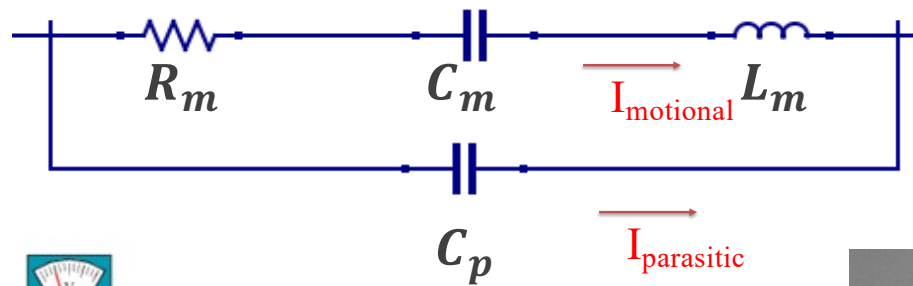
$$f \propto \sqrt{\frac{E n}{\rho t}}$$

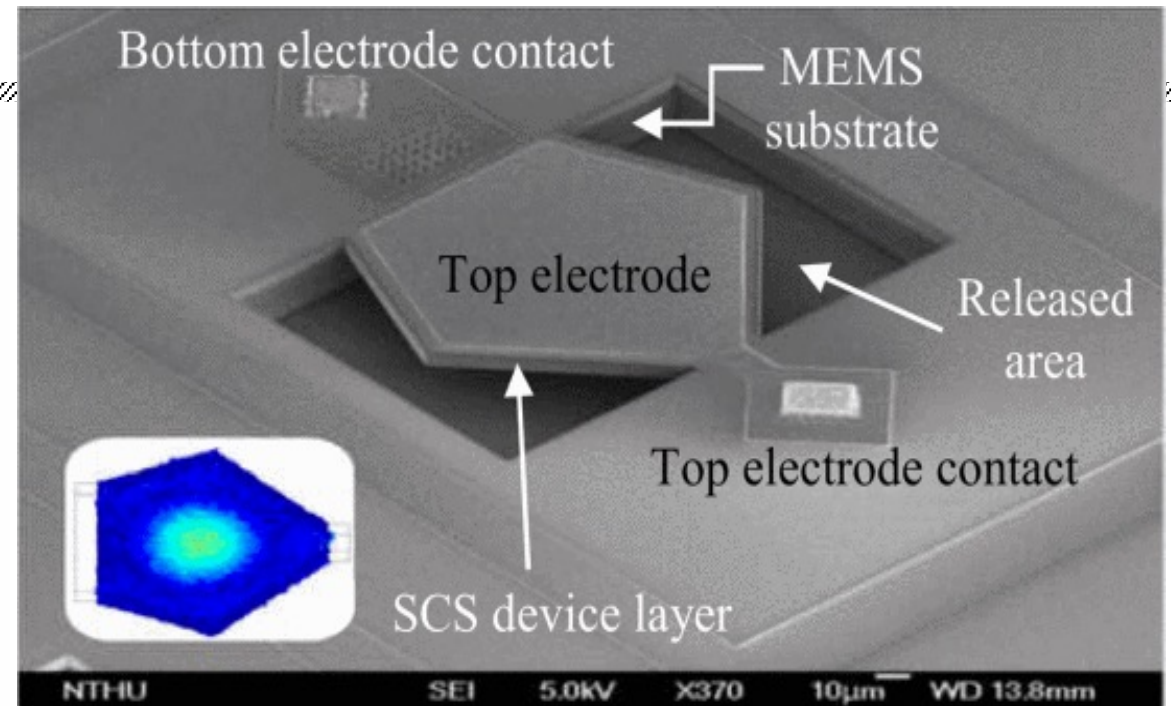
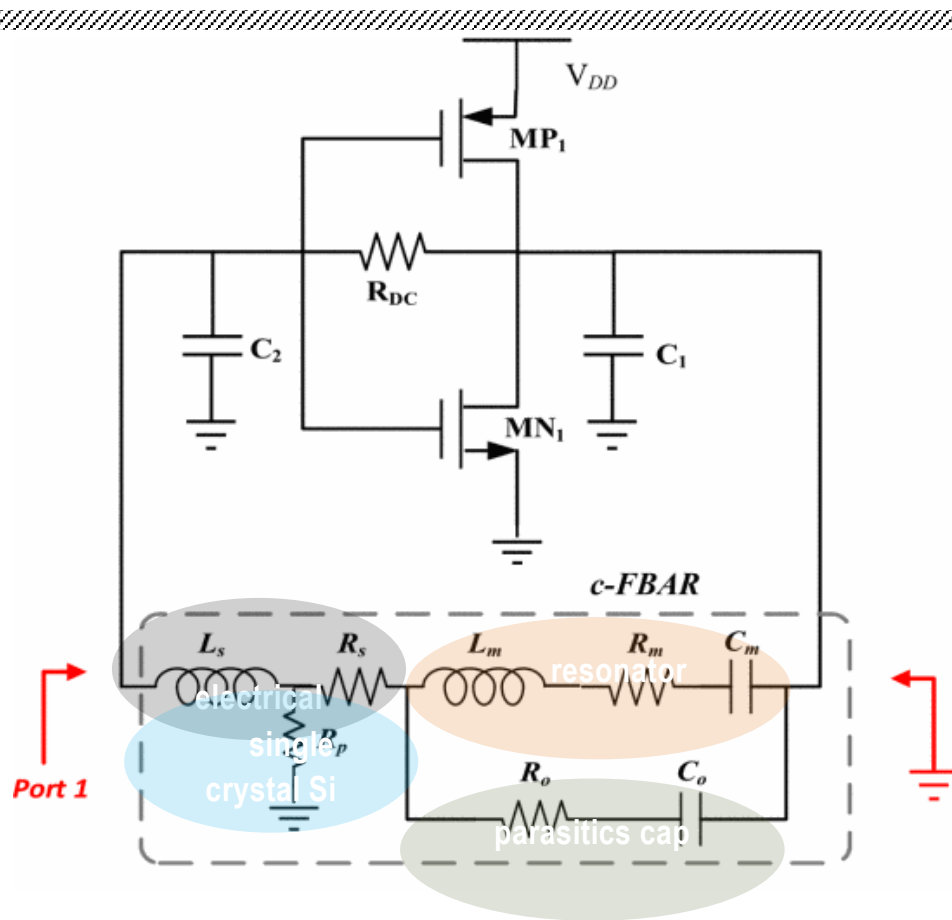


Equivalent circuit

- When the mechanical resonator moves
 - Motion is amplified around the resonance frequency (Q factor)
 - This is a mechanical system, modelled as an LCR in series: *motional impedance*
- But charges will also move according to the physical capacitor
 - Thus another path for the charges is created – *parasitic capacitance*

mechanical
electrical





Q=2000 at 3.25 GHz
Single crystal silicon on Piezo

G. Pillai, et al, "3-GHz BAW composite resonators integrated with CMOS in a single-chip configuration," *2016 IEEE International Frequency Control Symposium (IFCS)*, New Orleans, LA, 2016, pp. 1-4. doi: 10.1109/FCS.2016.7563593

Bulk Acoustic Wave (BAW) filter

- BAW technology relies on thin film deposition of piezoelectric aluminum nitride (AlN) on a silicon (Si) substrate.
- The bulk acoustic wave travels and resonates in the vertical direction so that the thin film thickness determines the resonant frequency.
- Two ways of confining acoustic energy within the thin film piezoelectric layer:
 - MEMS process to suspend AlN plate at a certain gap from the Si substrate, = Film Bulk Acoustic Resonator (FBAR)
 - Or make Bragg reflector to reflect most of the acoustic energy back into the piezoelectric AlN plate, known as a Solidly Mounted Resonator (SMR)

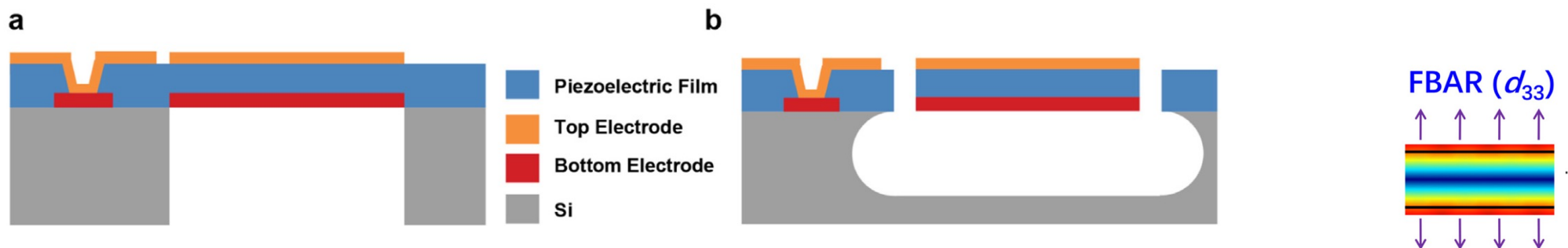


Fig. 9 | Cross-sectional drawing of FBAR device structures: a, membrane-type FBAR and b, airgap-type FBAR.

FBAR

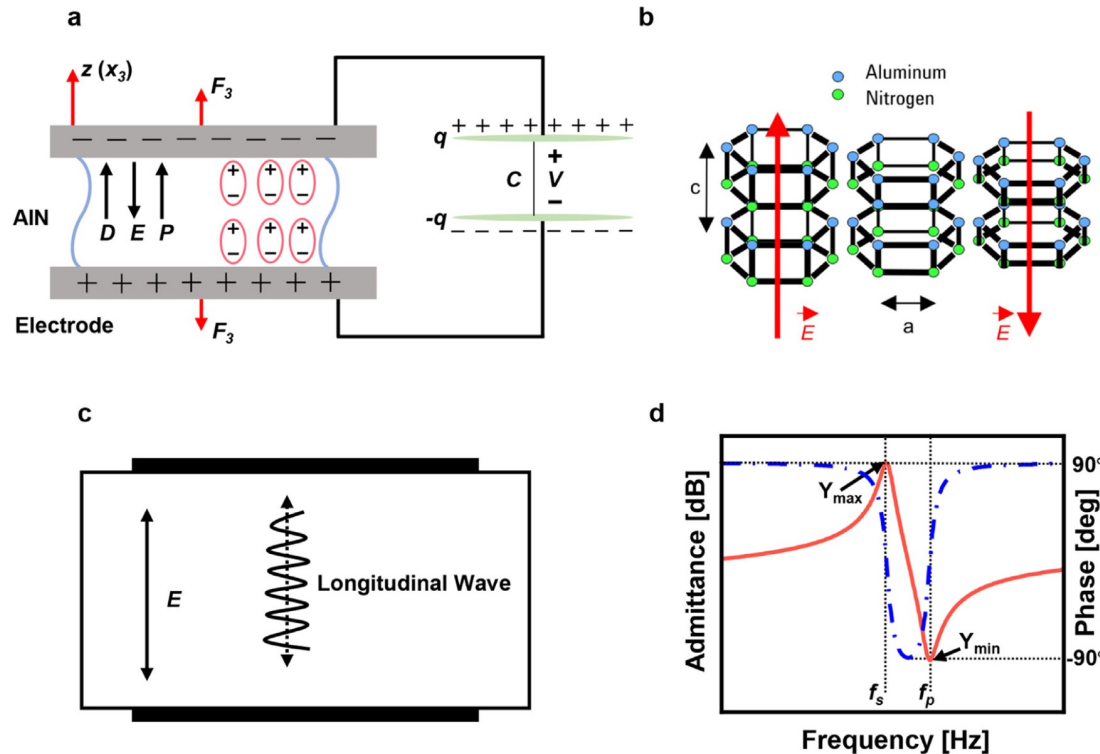


Fig. 10 | **a**, Electric displacement and polarization in an FBAR resonator. Reprinted with permission from ref.¹⁰⁵. © 2008, Universitat Autònoma de Barcelona. **b**, Deformation of the crystal structure when electric field is applied in the c -axis. Reprinted with permission from ref.¹⁰⁶. © 2001, Avagotech. **c**, The direction of electrical field and acoustic wave propagation in an FBAR resonator. **d**, Typical frequency response of an FBAR resonator.

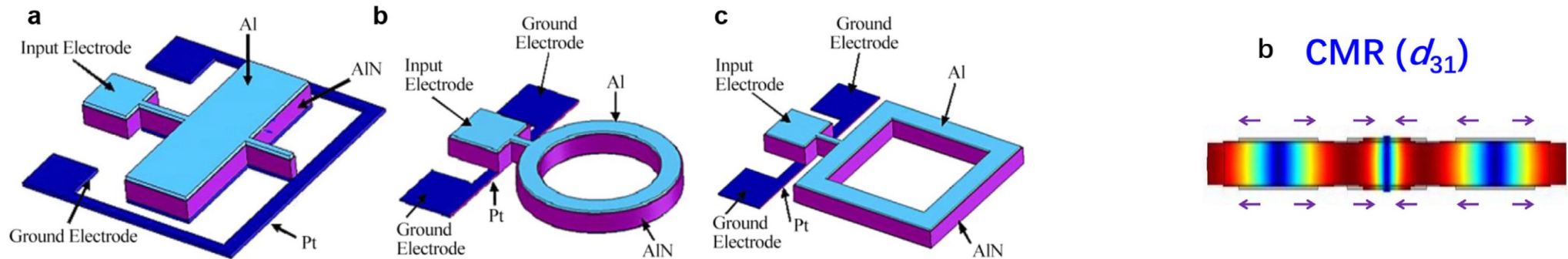
- The resonant frequency of FBAR resonator is determined by the total thickness of the metal-piezo-metal thin film layer stack.
- Only one frequency can be obtained on the same wafer or chip with a fixed layer stack.

The longitudinal piezoelectric coefficient (d_{33}) is primarily the mechanism responsible for FBAR operation: vibration and voltage are both in the z direction

$$f_{res} = \frac{v}{t}$$

- v acoustic velocity in the piezoelectric material,
- t thickness of the piezoelectric layer.

Contour-mode resonators (CMR): frequency is set by lateral dimensions



Three different designs of contour-mode resonators (CMR). a, Rectangular plate. b, Circular ring. c, Square-shaped ring. Reprinted with permission from [1], 2006, IEEE.

$$f_0 = \begin{cases} \frac{1}{2L} \sqrt{\frac{E_p}{\rho}} \\ \frac{1}{2W} \sqrt{\frac{E_p}{\rho}} \end{cases}$$

The transverse piezoelectric coefficient d_{31} is utilized to excite a mechanical vibration in the lateral (in plane) direction, while the voltage is applied in the z direction

- E_p equivalent Young's modulus of the metal-piezo-metal stack,
- ρ is the equivalent density
- L (or W) denotes the length (or width) of the rectangular plate depending on which of the length-extensional or width-extensional mode is used.

The resonant frequency of CMR can be easily set by lateral dimensions, so multi-frequency chips are possible